Financial Policy
Time Value of Money

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Course Outline

- **Introduction**
  - Lecture 1: Financial Management and the Business Environment
    - Readings: Brealey & Myers (B&M) Chap1, Chap2.
  - **Lecture 2: Time Value of Money**
    - **Readings: B&M Chap3**
- **Investment Decisions**
  - Lecture 3: Investment Appraisal Methods (**+ *Quizz 1***)
    - Readings: B&M Ch5
  - Lecture 4: Net Working Capital and Cash Flow Management
    - Readings: B&M Ch30, 31.
  - Lecture 5: Financial Forecasting and Budgeting
    - Readings: B&M Ch6, 19.2, 29.
  - ***Week 6 Midterm Exam (1h30)***
- **Financing Decisions**
  - Lecture 6: The Value of Bonds and Common Stocks
    - Readings: B&M Ch4.
  - Lecture 7: Internal Funds, Equity Financing and Dividend Policy
    - Readings: B&M Ch14, 15, 16.
  - Lecture 8 and 9:Capital Structure and the Cost of Financing (**+ *Quizz 2***)
    - Readings: B&M Ch9, 10.1, 19.
  - Lecture 10: Overall Recap
  - ***Week 11 Final Exam (3h)***

Class Outline

- Time Value of Money
- Simple and Compound Interest
- Present Value
- Annuities
- Perpetuities

Time Value of Money (recap)

- Time is money !
- Why?
  - Why do you prefer $10,000 now instead of receiving $10,000 in a year?
- Choosing between
  - Consumption (spending) today
  - Consumption in a year
  - Tomorrow is another day…
    - Future is not certain

Time Value of Money (recap)

- What's the time value of money ?
- To compensate for
  - Delayed consumption
  - Risk
    - Inflation Risk
    - Risk Premium

Time Value of Money (recap)

- The basic problem faced by financial manager is
  - I have to **spend money today** (for instance to build a plant), that will **generate cash flows in the future**
- How to value future cash flows ?
- Does it make sense to commit money today for revenue tomorrow ?

Time Value of Money (recap)

- What determines trade-off between
  - Current dollars *(or euros, pounds, pesos,…)*

    and
  - Future dollars

INTEREST RATE

Time Value of Money (recap)

- The interest rate is the price/reward you ask for accepting not to dispose immediately of your cash
- The interest is the cost charged by the lender to the borrower, over a period *t*, at a rate *r*
- Interest can be
  - Simple
  - Compound

Simple Interest

- When we have:
  - **Only one period**
  - **Only one cash flow**
- => **Simple interest**
- Simple interest C
  - P = Principal in €, $, £, etc.
  - t = duration, in years, months or days
  - r = interest rate per year in %

Simple Interest

- Interest earned: $C = P \times r \times t$; ***r: is an annual rate***
  - When *t* duration in years $C = P \times r \times t$
  - When *t* duration in months $C = P \times r \times \frac{t}{12}$
  - When *t* duration in days $C = P \times r \times \frac{t}{360}$
- **Future value** FV = $P + C = P \times (1 + r \times t)$

Simple Interest

- Examples
  - An amount of €10,000 is invested at a rate of 10% over 6 months. What's the amount of the earned interest ?
  - An amount of €20,000 is invested at a rate of 12% over 4 months and 15 days. What's the amount of the earned interests ?
  - What's the **future value** of an amount of €45,000 invested at 8% over 6 months and 20 days ?

Compound Interest

- When
  - **There is more than one period**
  - **Several cash flows**
- => **Compound Interest**
- The interest earned by the principal after one period are added to the initial principal to make a **new beginning principal** for the following year
- Interests are said to be **compounded**

Compound Interest

- Example : €10,000 invested at 10%, coupons + principal to be redeemed after 3 years
  - Interests for the first year
    - 10,000 x 10% = €1,000
    - Interests are added to the initial principal, to start with a new beginning principal for the second year: 10,000 + 1,000 = €11,000
  - Interests for the second year
    - 11,000 x 10% = €1,100
    - New beginning principal for the third year: €12,100
  - Interests for the third year
    - 12,100 x 10% = €1,210
  - After 3 years the borrower must repay
    10,000 + 1,000 + 1,100 + 1,210 = €13,310

Compound Interest

- I invest *P* for *n* years at *r* per year
  - At the end of the year 1
    - I have $FV_1 = P \times (1 + r)$ which is my beginning principal for year 2
  - At the end of the year 2, I will have FV_2
    - $FV_2 = FV_1(1+r) = P(1+r)(1+r) = P(1+r)^2$

      ⋮
  - At the end of the year *n*, I will have FV_n
    - $FV_n = FV_{n-1}(1+r) = P(1+r)(1+r)\ldots(1+r) = P(1+r)^n$
- FV of principal P, at the end of n years is

$$\boldsymbol{FV_n = P\,(1+r)^n}$$

Compound Interest

- Example
  - A principal of €50,000 is invested at an annual compound rate of r = 9%. What is the future value after 7 years?

Compound Interest

- Future value with non-annual periods (month, quarter, etc.)
  - The previous formula is still valid if the interest rate and the duration are expressed in the same time unit as the period itself.
- Example
  - A principal of €10,000 is invested with a quarterly compounding rate of 2.5%. What is the FV after 6 years?

Compound Interest

- What if the period is not a round number?
  - It must be expressed as a fraction of year
- Example
  - A principal of €20,000 is invested at an annual compounding rate of 11% over 7 years and 3 months. What is its future value?

Compound Interest

- Problem 1
  - P = €20,000
  - Annual compounding
  - Duration $n = 7$ years
  - $r = 9.5\%$
  - FV_7 ?

Compound Interest

- Problem 2
  - P = €30,000
  - Annual compounding
  - Duration n = 11 years
  - Future value FV_{11} = €89,971.77
  - Annual interest rate?

Compound Interest

- Problem 3
  - P = 40 000€
  - Semi-annual compounding
  - Future value FV_n = €76,597.84
  - Semi-annual interest rate r = 4.75%
  - Duration n = ?

Compound Interest

- Problem 4
  - Annual compounding
  - Duration n = 10 years
  - Future Value FV_{10} = €123,661.92
  - Annual interest rate r = 7.5%
  - Principal initially invested ?

Compound Interest

- Different securities may have different compounding periods
  - For example, in US and Britain coupon bonds pay semiannual interest while in France and Germany they pay annual interest
- What if we want to compare securities with different compounding periods?
- To compare compounded securities we need to compare rates over the same length of time (usually one takes one year)

Compound Interest

- Compare securities in term of equivalent annually compounded rate (EAR)

EAR $=(1+r_m)^m - 1$

  - m is the number of compounding period in a year
  - r_m is the sub-period rate.
  - ***NB: Generally, when the « quoted » annual rate (let's call it r) is given, to compute the EAR, you must first compute the sub-period rate:***

$$r_m = \frac{r}{m}$$

  - ***r is a proportional rate (based on simple interest rate calculation). It is compounded m times per year.***

Compound Interest

- Example of Equivalent Annual Compounding Rate
  - A car loan charges interest at 1% per month. The car seller announces an annual percentage rate of **12%** per year
  - But the EAR = $(1+.01)^{12} - 1$ = **12.6825%** per year
  - This is the rate you actually pay

Compound Interest

- Equivalent Rate
  - Example 1 : compute the semiannual rate r_2 equivalent to the annual rate r = 9.5%
  - Example 2 : compute the monthly rate r_{12} equivalent to the quarterly rate r_4 = 7%
  - Example 3 : compute the semiannual rate r_2 equivalent to the monthly rate i_{12} = 1,5%

Compound Interest

- Continuous Compounding
  - Sometimes it may be reasonable to assume that cash flows occur uniformly throughout the year
  - For example when you estimate sales over the year
- When the compounding period tends to zero, *m* the number of compounding periods increases without limit
- In continuous time EAR = $\underset{m\to\infty}{\text{Lim}}\left(1+r_m\right)^m - 1 = e^r$
- €100 invested at a **continuous interest rate** *r* for *t* years becomes €100e^{rt}

Compound Interest

- Continuous Compounding
  - Example : a given initial principal was multiplied after a year by 1.10.
  - Compute the corresponding continuous and discret rates.

Present Value

- To find the **present value PV** is to **discount** a future value FV back to the present
- Discounting is the opposite of compounding

$$PV_0 = \frac{FV_t}{(1+r)^t}$$

Present Value

- Present value of multiple cash flows occurring at different times

  - Present value of
    - PV(C_1) = C_1 / $(1+r)^1$
    - PV(C_2) = C_2 / $(1+r)^2$
    - **...**
    - PV(C_n) = C_n / $(1+r)^n$
- We can add up all the present values of the individual cash flows

Present Value

- Discounted Cash Flows Equation **DCF**

$$PV_0 = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_n}{(1+r)^n}$$

$$PV_0 = \sum_{i=1}^{n} \frac{C_i}{(1+r)^i}$$

Present Value

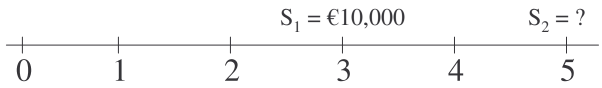
- Equivalent Present Value
  - Two series of cash flows are said to be equivalent if their present value at the same discount rate is equal

Present Value

- Equivalent Present Value
  - Exemple: I have to compare between receiving P_1 in 4 years and P_2 in 7 years
    - P_1 (Principal) = €68,024.45
    - Maturity = 4 years
    - Annual rate = 8%
    - PV = 68,024.45 x $1{,}08^{-4}$ = €50,000
    - P_2 (Principal) = €85,691.20
    - Maturity = 7 years
    - Anuual rate = 8%
    - PV = €85,691.20 x $1{,}08^{-7}$ = €50,000
  - At the discount rate of 8%, €68,024.45 in 4 years is equivalent to €85,691.20 in 7 years

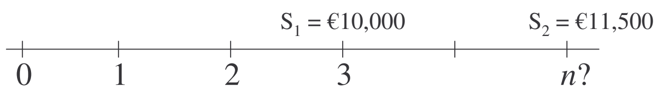
Present Value

- Pb-1 : Compute the principal amount
  - You wish to replace a settlement of S_1=€10,000 initially planned in 3 years by another settlement of S_2 in 5 years. The current annual interest rate is 8%. How much must you offer ?



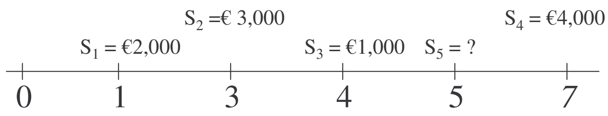
Present Value

- Pb-2 : Compute a date
  - You wish to replace a settlement of S_1=€10,000 initially planned in 3 years by another settlement of €11,500. The current annual interest rate is 6%. When should this new settlement occur?



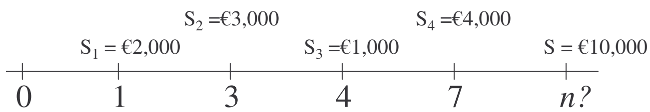
Present Value

- Pb-3 : Compute a principal amount
  - You wish to replace 4 settlements
    - S_1 = €2,000 in a year
    - S_2 = €3,000 in 3 years
    - S_3 = €1,000 in 4 years
    - S_4 = €4,000 in 7 years
  - By a sole settlement S_5 in 5 years. Compute S_5 (the annual rate is 7%).



Present Value

- Pb-4 : Compute a settlement date
  - You wish to replace 4 settlements
    - S_1 = €2,000 in a year
    - S_2 = €3,000 in 3 years
    - S_3 = €1,000 in 4 years
    - S_4 = €4,000 in 7 years
  - By a sole settlement of €10,000. When must this settlement occur (the annual rate is 7%)?



Present Value

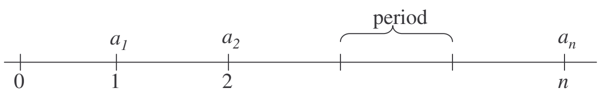
- Pb-5 : Comparison of cash flows
  - 1°)Is it better to receive €80,000 in 2 years or €105,000 in 5 years (the annual interest rate is 8,5%)?
  - 2°)What is the interest rate *r* that would make equivalent these two cash flows?

Present Value

- Exercise 1
  - A €100,000 loan is redeemed in 10 annuities. The 2 first annuities are worth Y, the 4 next annuities are 2Y and the last annuities are 3Y. Compute Y (the annual interest rate is12%).
- Exercise 2
  - You owe €10,000, €20,000, €30,000 in resp. 1, 2 and 3 years. In fact you would rather repay in two equal amounts in 4 and 5 years At the prevailing 10% interest rate, how much should you repay?

Annuities

- Definition:
  - Annuities are a series of cash flows occurring at constant interval period for a given time
- A series of annuities is defined by :
  - Date of the first cash flow
  - Period = **constant duration** between two cash flows
  - Number of cash flows *n*
  - Amount of each cash flow a_1, a_2, …, a_n



Annuities

- The aim of a series of annuities is generally to
  - Build up a capital
    - Investment annuities
  - To service a debt (interest + principal)
    - Amortization annuities (to reimburse a debt)

Constant Annuities

- Future value of a series of constant annuities

Annuities	Date of the cash flows	Number of compounding periods	Future value of the annuities
a	1	(n-1) period	$a(1+r)^{n-1}$
a	2	(n-2) period	$a(1+r)^{n-2}$
a	3	(n-3) period	$a(1+r)^{n-3}$
a	n-1	1 period	$a(1+r)^{1}= a\ (1+r)$
a	n	0 period	$a(1+r)^{0} = a$
			FV

$$FV = \sum_{j=1}^{n} a(1+r)^{n-j} = a\sum_{j=1}^{n}(1+r)^{n-j} = a\,\frac{(1+r)^{n}-1}{r}$$

Constant Annuities

- Example 1- Compute the future value
  - Compute the future value of a series of 15 constant annuities each worth €10,000. Compounding rate of 8,5%.

Constant Annuities

- Example 2-Compute the constant annuity
  - We are on Apr 15, 2000
  - In order to build up a capital of €1,000,000 as of April 15, 2011, you plan to set aside a constant amount of money on a bank account yielding 10%.
    - Date of the first settlement : April 15, 2001
    - Last settlement : April 15, 2011
  - How much should you save each year ?

Constant Annuities

- Example 3-Compute the number of annuities
  - Saving every year €20,000, with a compounding rate of 7.5%, you wish to build a capital of €200,000 at the end of the last annuity.
  - How many annuities will it take (round number)?

Constant Annuities

- Example-4
  - Compute the future value of 84 monthly payments of €1000 each. Annual rate of 10%.

Constant Annuities

- Present value of a series of constant annuities

Annuities	Date of the cash flows	Number of discounting periods	Present value of the annuities
a	1	1 period	$a\ (1+r)^{-1}$
a	2	2 period	$a\ (1+r)^{-2}$
a	3	3 period	$a\ (1+r)^{-3}$
a	n-1	(n-1) period	$a\ (1+r)^{-(n-1)}$
a	n	n period	$a\ (1+r)^{-n}$
			PV

$$PV = \sum_{j=1}^{n} a(1+r)^{-j} = a\sum_{j=1}^{n}(1+r)^{-j} = a\,\frac{1-(1+r)^{-n}}{r}$$

Constant Annuities

- Pb1- Compute the PV of a series of 15 constant annuities of €1000 each. Discount rate = 8%.

Constant Annuities

- Pb2- A series of 10 constant annuities is discounted at 10,5%. Its PV is worth €200,000. What's the amount of each annuity ?

Constant Annuities

- Compare a series of 10 annuities of €2000 each paid from April 15, 2003 and a series of 14 annuities of €1500 each paid from April 15, 2002. Discount rate 9%.

Constant Annuities

- Relationship between Future Value and Present Value
  - Future Value $FV = a\frac{(1+r)^n - 1}{r}$
  - Present Value $PV = a\frac{1-(1+r)^{-n}}{r}$

$FV = PV(1+r)^n$ and $PV = FV(1+r)^{-n}$

Perpetuities

- Definition
  - Series of equal cash flows at end of successive periods continuing forever
- Present value of a perpetuity

$$PV = \lim_{n \to \infty} a \frac{1-(1+r)^{-n}}{r} = \frac{a}{r}$$