CERAM Sophia Antipolis
European School of Business

A Fractal View of Risk

Sophia Antipolis, 16th December 2005

MUGAAS Annemarta Strand, SINDONEN Natalia, TONGSEEDANG Petcharat,
JORCH Helge, UZSOKI David

Agenda

A Fractal View of Risk

- **Problems of the Classic Models**
- **Introduction to the Fractal Concept**
- **Fractal Geometry**
- **Fractals in Finance**
- **Conclusion**
- **Sources**

Agenda

A Fractal View of Risk

- **Problems of the Classic Models**
- **Introduction to the Fractal Concept**
- **Fractal Geometry**
- **Fractals in Finance**
- **Conclusion**
- **Sources**

Problems of the Classic Models

Analyzing Investments

- Most common tool: volatility and beta (only make sense if prices vary by the bell curve)
- No clear pattern has yet emerged to explain why each currency has the value it does
- Different studies have different Hurst exponent values without any pattern among them

Building Portfolios

- Modern portfolio theory: managers construct portfolios based on the market risk premium and the beta of the security (CAPM)
- Conventional models consistently understate beta
- "Building a portfolio today is rather statistics than intelligence": managers use Monte Carlo simulation which is far from perfect

Four Essential Problems

Valuing Options

- Unrealistic assumptions of Black-Scholes Formula: prices vary by the bell curve and do not jump, taxes and commissions do not exist and volatility does not change through the life of the option
- The last assumption is the biggest flaw: implied volatility differs significantly from the volatility that actually occurred

Managing Risk

- The standard method is Value at Risk (VaR) which relies on the Brownian motion
- It is flawed because it significantly underestimates extreme risk. There is no limit to how much you can lose
- Banks: the losses could be even bigger than their own capital

Agenda

A Fractal View of Risk

- Problems of the Classic Models
- **Introduction to the Fractal Concept**
- Fractal Geometry
- Fractals in Finance
- Conclusion
- Sources

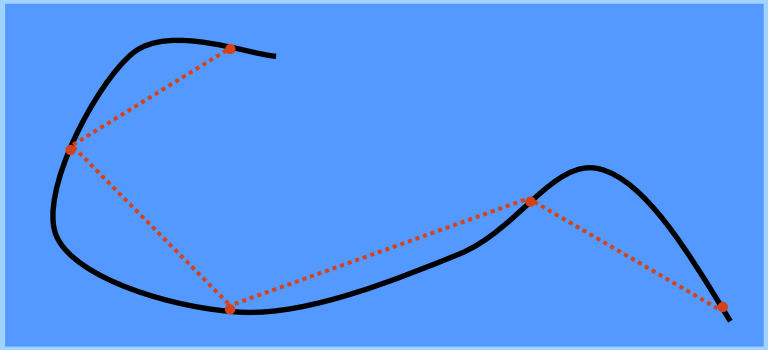
Introduction to the Fractal Concept

Introduction

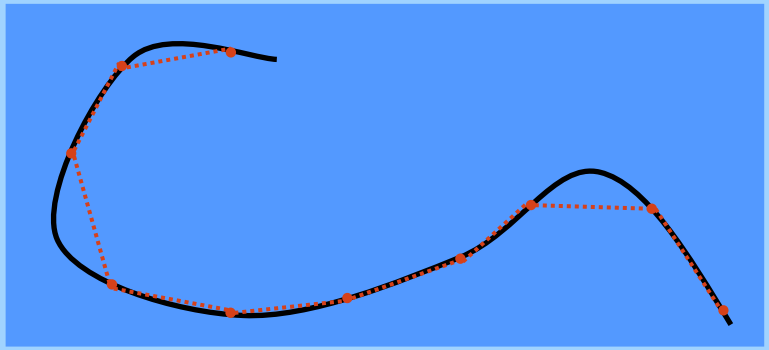
- **Invented by Benoit Mandelbrot by using the concept of finding order in seemingly irregular systems, so called “Fractal Geometry”**
- **The reason for introducing fractals is motivated by the roughness of nature, which is difficult to describe with classical geometry**

Example:

The approximate length of the curve is 4X if the yardstick length is X, and 9 * X/2 = 4.5 X if the yardstick length is X/2.



Yardstick length X

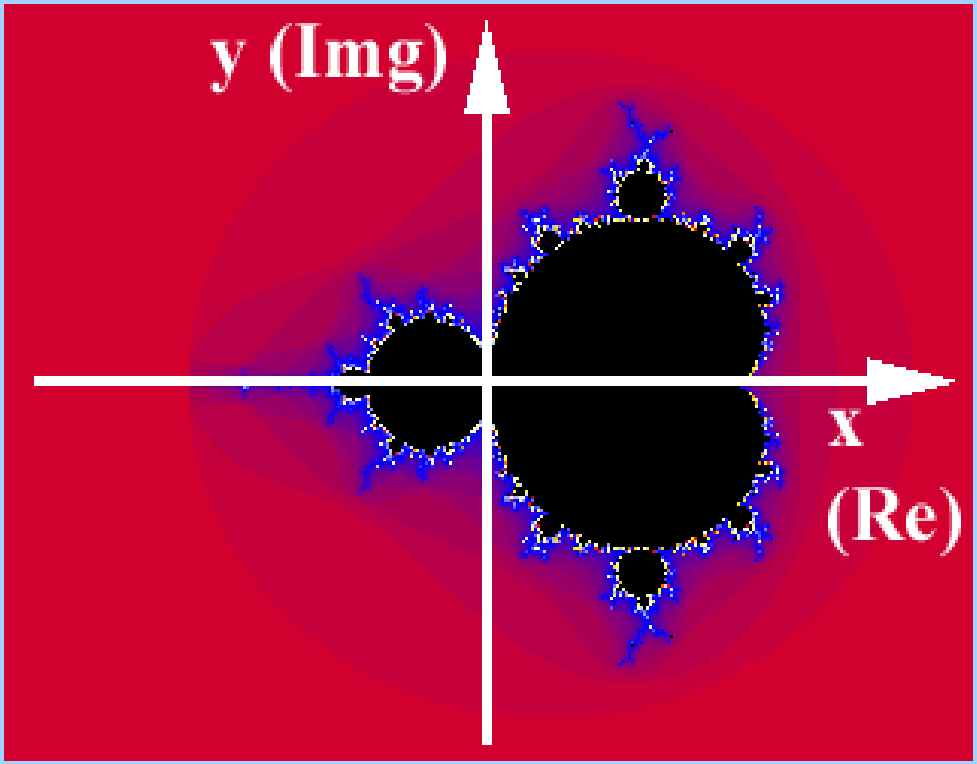


Yardstick length X/2

Introduction to the Fractal Concept

Definition

- **Geometrical entities characterised by basic patterns that are repeated at ever decreasing sizes**
- **They are relevant to any system involving self-similarity repeated on diminished scales**
- **There is no strict mathematical definition of fractal sets, but some properties expected to be seen in a fractal set are:**
  - Some kind of self-similarity
  - Fine structure on all scales
  - High degree of irregularity
  - Non-integer dimension



Branches of trees, florets of cauliflowers and bifurcations of rivers are examples of natural fractals

Introduction to the Fractal Concept

Examples

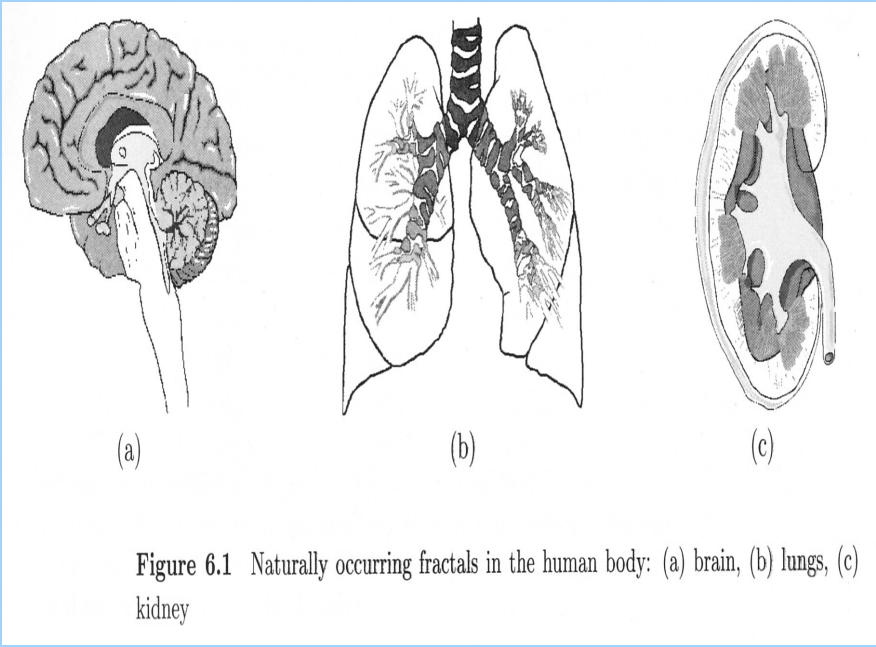


Figure 6.1 Naturally occurring fractals in the human body: (a) brain, (b) lungs, (c) kidney

Agenda

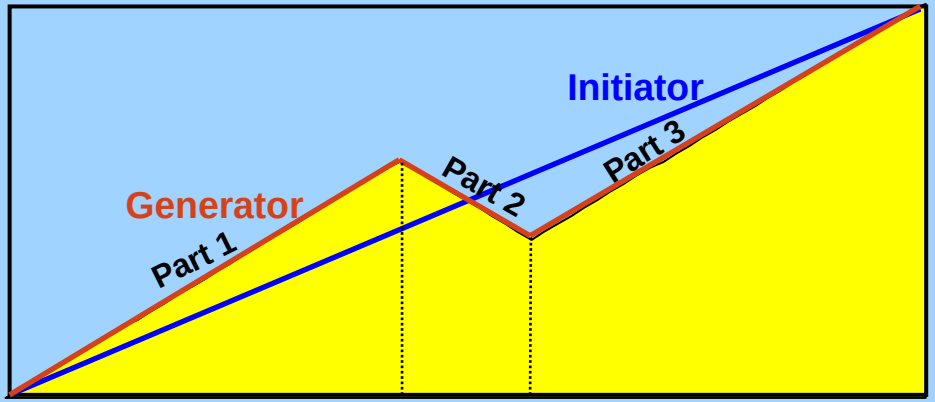
A Fractal View of Risk

- Problems of the Classic Models
- Introduction to the Fractal Concept
- **Fractal Geometry**
- Fractals in Finance
- Conclusion
- Sources

Fractal Geometry

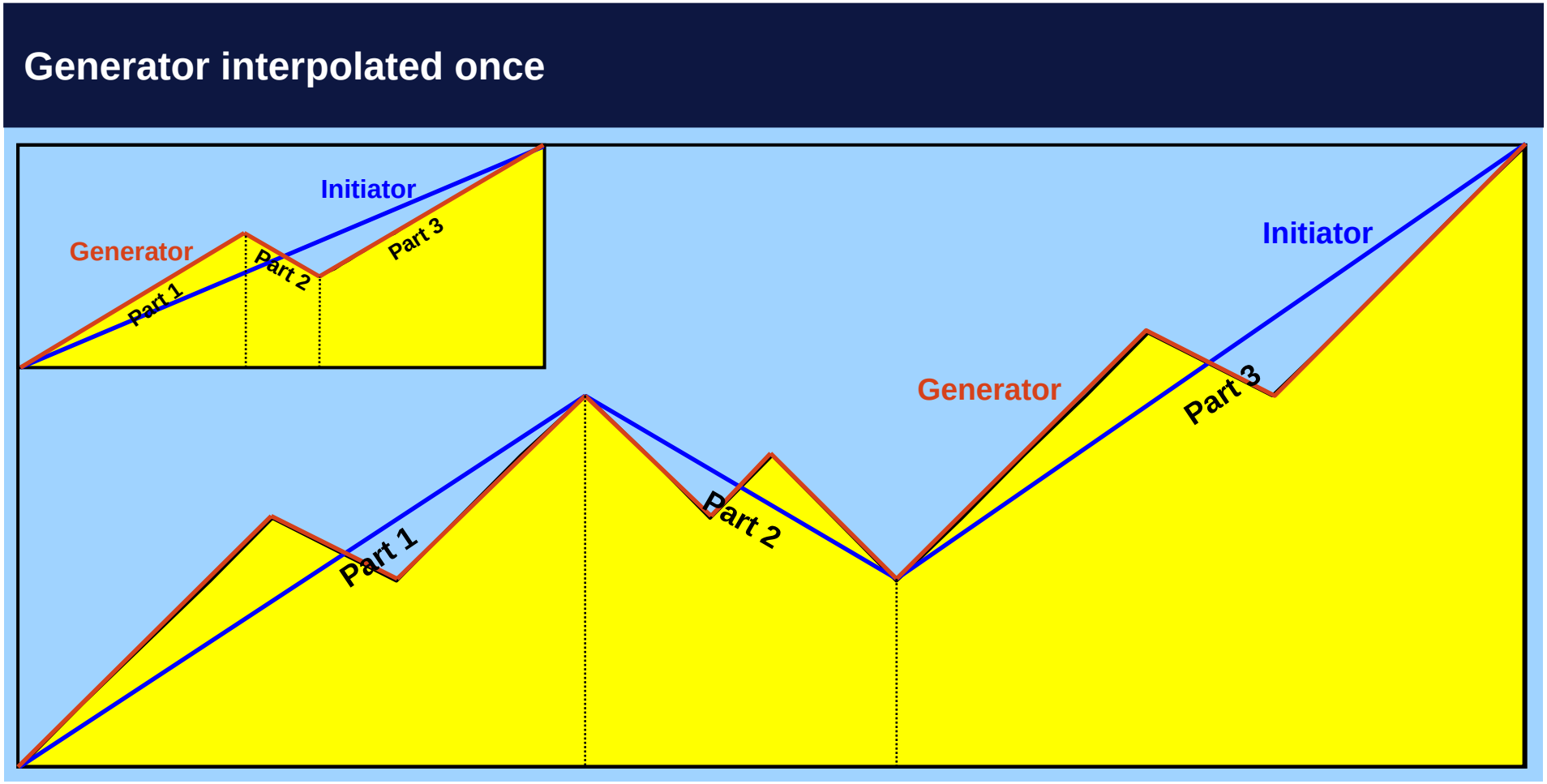
Initiator and Generator

- **Fractal consists of**
  - Initiator
  - Generator
- **Process begins with a price represented by a straight trend line called "initiator"**
- **A broken line called "generator" is used to create the pattern that corresponds to a slow up and down price fluctuation**

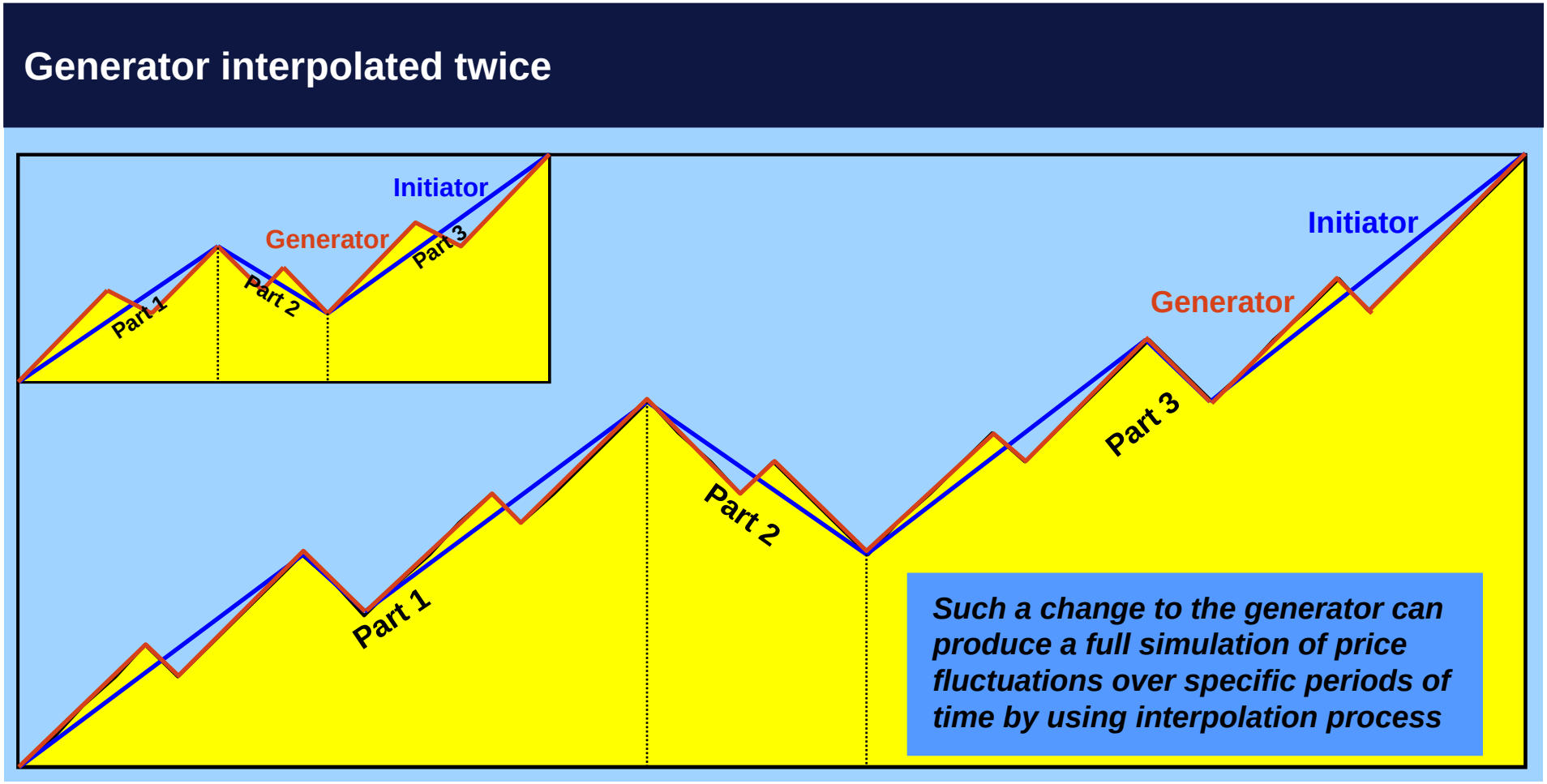


- **Powerlaw and self similarity (as shown in example 1)**
- **To create a multifractal from a unifractal, the key step is to lengthen or shorten the horizontal time axis so that the pieces of the generator are either stretched or squeezed. At the same time, the vertical price axis may remain untouched (as shown in example 2)**

Fractal Geometry



Fractal Geometry



Fractal Geometry

Applied in practice

- These finding suggests that a fractal generator can be developed based on historical market data
- The actual model used does not simply inspect what the market did yesterday or last week
- It is more realistic illustration of market fluctuations, called fractional Brownian motion
- The charts created from the generators produced by this model can simulate alternative scenarios based on previous market activity

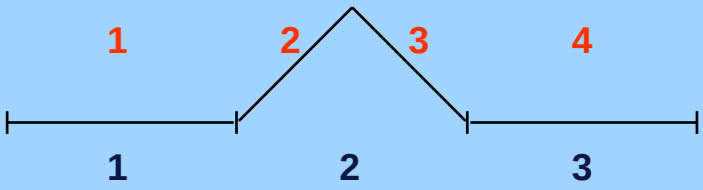
Fractal Geometry

Fractal Dimensions

- Fractal dimension measurement is applied for nonlinear dynamics
- Fractal dimension considers fluctuation structure, which standard deviation fails to represent
- Fractal Dimension D_B=1 for a straight line
- Fractal Dimension D_B=2 for a square

$$D_B = \lim_{\varepsilon \to 0} \frac{\log N}{\log(1/\varepsilon)}$$

Example:



$$D_s = \frac{\log 4}{\log(1/3)} = 1.2618$$

Agenda

A Fractal View of Risk

- Problems of the Classic Models
- Introduction to the Fractal Concept
- Fractal Geometry
- **Fractals in Finance**
- Conclusion
- Sources

Mandelbrot's model aims to predict volatility, so the goal is to manage risk and to sidestep the worst kicks from the market, i.e. not to make money

Fractals in Finance

Mandelbrot's Model

How it started

- First studied cotton prices over a large period of time
- Seemingly no pattern in how the prices moved
- After many years of study, Mandelbrot found that the cotton prices followed a scaling law
- This was the first step towards self-similarities:
  the prices showed similar patterns at different time intervals, which is the principle of fractals

Implications & Assumptions

- Agrees with the Martingale condition in that prices are difficult to predict
- Financial markets are turbulent like the wind, and that volatility itself is volatile. Thus financial markets are a lot more risky than other models assume, because prices move around a lot more than what seems to fit into a Gaussian curve
- The model mimics how much a price varies; it does not trace the real path of the prices, but will behave statistically in the same way, thus it would be able to predict future volatility and more efficiently manage risk
- Prices jump or fall abruptly, rather than glide up and down continuously, showing that volatility clusters in time, indicating that there might be some pattern
- Bubbles are inevitable and we cannot predict it in any useful way
- There is dependence among price moves, although not necessarily correlation. Thus any major changes can be predicted, but not the direction of them. This dependence is referred to as the "Joseph Effect"

Agenda

A Fractal View of Risk

- Problems of the Classic Models
- Introduction to the Fractal Concept
- Fractal Geometry
- Fractals in Finance
- **Conclusion**
- Sources

The model is generally accepted amongst experts but needs to be developed further to be applicable to financial markets

Conclusion

Summary

- Mandelbrot applied the fractals he found in nature to the financial markets
- His goal was to find a model that would better fit the market than what is applied today, as he believes that markets are a lot more risky than assumed and that we are in need of better measures of risk
- His model does not aim or claim to predict prices, but rather to forecast volatility

Conclusion

- There is general consensus that Mandelbrot is right in his view on market movements, but the fractal model is not widely used
- The model needs to be developed further before it can be of any major practical use, and we look forward to someone taking up the thread and develop it further…

Agenda

A Fractal View of Risk

- Problems of the Classic Models
- Introduction to the Fractal Concept
- Fractal Geometry
- Fractals in Finance
- Conclusion
- Sources

Sources

Books

- **Bernoit B. Mandelbrot**
  The (Mis)behavior of Markets
- **Bernoit B. Mandelbrot**
  Fractals and Scaling in Finance
- **Michael M. Dacorogna, Ramazan Gencay, Ulrich Müller, Richard B. Olsen, Olivier V. Pictet**
  An Introduction to High-Frequency Finance
- **Edgar E. Peters**
  Chaos and Order in the Capital Markets
- **Edgar E. Peters**
  Fractal Market Analysis

Others

- **Software: Trusoft, Benoit™ 1.3**
  Fractal Analysis System
- **Bernoit B. Mandelbrot**
  A Multifractal Walk Down Wall Street
  Article published on http://www.elliottwave.com/education/SciAmerican/Mandelbrot_Article2.htm
- **Michel H. Bouchet**
- **David E. Hampton**

For further details, please contact:
natalia.sindonen@ceram.fr